LISTA 06_7 EQUAÇÕES DIFERENCIAIS
Sistema de equações lineares não homogêneas
Respostas no final
Gabaritos na página do professor

Em cada um dos problemas de 1 a 12, encontre a solução geral do sistema de equações dado.

1. $$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} e^t \\ t \end{pmatrix}$$

2. $$\mathbf{x}' = \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\mathbf{x} + \begin{pmatrix} e^t \\ \sqrt{3}\,e^{-t} \end{pmatrix}$$

3. $$\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} -\cos t \\ \operatorname{sen} t \end{pmatrix}$$

4. $$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} e^{-2t} \\ -2e^t \end{pmatrix}$$

5. $$\mathbf{x}' = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix}\mathbf{x} + \begin{pmatrix} t^{-3} \\ -t^{-2} \end{pmatrix}, \qquad t > 0$$

6. $$\mathbf{x}' = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}\mathbf{x} + \begin{pmatrix} t^{-1} \\ 2t^{-1} + 4 \end{pmatrix}, \qquad t > 0$$

7. $$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^t$$

8. $$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t$$

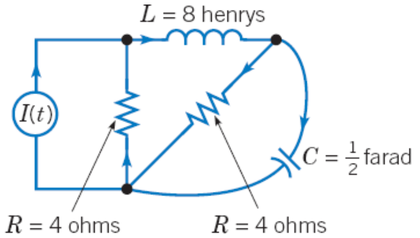
9. $$\mathbf{x}' = \begin{pmatrix} -\frac{5}{4} & \frac{3}{4} \\ \frac{3}{4} & -\frac{5}{4} \end{pmatrix}\mathbf{x} + \begin{pmatrix} 2t \\ e^t \end{pmatrix}$$

10. $$\mathbf{x}' = \begin{pmatrix} -3 & \sqrt{2} \\ \sqrt{2} & -2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$

11. $$\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 \\ \cos t \end{pmatrix}, \qquad 0 < t < \pi$$

12. $$\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} \csc t \\ \sec t \end{pmatrix}, \qquad \frac{\pi}{2} < t < \pi$$

13. O circuito elétrico mostrado na figura é descrito pelo sistema de equações diferenciais abaixo em que x_1 é a corrente através do indutor, x_2 é a queda de tensão através do capacitor, e *I(t)* é a corrente fornecida pela fonte externa.



$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{8} \\ 2 & -\frac{1}{2} \end{pmatrix}\mathbf{x} + \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix} I(t).$$

(a) Determine uma matriz fundamental $\boldsymbol{\Psi}(t)$ para o sistema homogêneo associado). Veja o Problema 25 da aula 06_4.

(b) Se $I(t) = e^{-t/2}$, determine a solução do sistema que satisfaz a condição inicial $\mathbf{x}(0) = \mathbf{0}$.

Em cada um dos Problemas 14 e 15, verifique se o vetor dado é a solução geral do sistema homogêneo associado e depois resolva o sistema não homogêneo. Suponha que t > 0.

14. $t\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 - t^2 \\ 2t \end{pmatrix}, \qquad \mathbf{x}^{(c)} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} t^{-1}$

15. $t\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -2t \\ t^4 - 1 \end{pmatrix}, \qquad \mathbf{x}^{(c)} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} t^{-1} + c_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} t^2$

16. Seja $x = \phi(t)$ a solução geral de $x' = P(t)x + g(t)$ e seja $x = v(t)$ uma solução particular do mesmo sistema. Considerando a diferença $\phi(t) - v(t)$, mostre que $\phi(t) = u(t) + v(t)$, em que $u(t)$ é a solução geral do sistema homogêneo $x' = P(t)x$.

17. Considere o problema de valor inicial

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{g}(t) \qquad \mathbf{x}(0) = \mathbf{x}^0$$

(a) Depois de olhar o Problema 15(c) na aula 06_5, mostre que

$$\mathbf{x} = \boldsymbol{\Phi}(t)\mathbf{x}^0 + \int_0^t \boldsymbol{\Phi}(t-s)\mathbf{g}(s)\,ds.$$

(b) Mostre, também, que

$$\mathbf{x} = \exp(\mathbf{A}t)\mathbf{x}^0 + \int_0^t \exp[\mathbf{A}(t-s)]\mathbf{g}(s)\,ds.$$

RESPOSTAS

1. $\mathbf{x} = c_1\begin{pmatrix}1\\1\end{pmatrix}e^t + c_2\begin{pmatrix}1\\3\end{pmatrix}e^{-t} + \frac{3}{2}\begin{pmatrix}1\\1\end{pmatrix}te^t - \frac{1}{4}\begin{pmatrix}1\\3\end{pmatrix}e^t + \begin{pmatrix}1\\2\end{pmatrix}t - \begin{pmatrix}0\\1\end{pmatrix}$

2. $\mathbf{x} = c_1\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}e^{2t} + c_2\begin{pmatrix}1\\-\sqrt{3}\end{pmatrix}e^{-2t} - \begin{pmatrix}2/3\\1/\sqrt{3}\end{pmatrix}e^t + \begin{pmatrix}-1\\2/\sqrt{3}\end{pmatrix}e^{-t}$

3. $\mathbf{x} = c_1\begin{pmatrix}5\cos t\\2\cos t + \operatorname{sen} t\end{pmatrix} + c_2\begin{pmatrix}5\operatorname{sen} t\\-\cos t + 2\operatorname{sen} t\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}t\cos t - \begin{pmatrix}1\\0\end{pmatrix}t\operatorname{sen} t - \begin{pmatrix}1\\1\end{pmatrix}\cos t$

4. $\mathbf{x} = c_1\begin{pmatrix}1\\-4\end{pmatrix}e^{-3t} + c_2\begin{pmatrix}1\\1\end{pmatrix}e^{2t} - \begin{pmatrix}0\\1\end{pmatrix}e^{-2t} + \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix}e^t$

5. $\mathbf{x} = c_1\begin{pmatrix}1\\2\end{pmatrix} + c_2\left[\begin{pmatrix}1\\2\end{pmatrix}t - \frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix}\right] - 2\begin{pmatrix}1\\2\end{pmatrix}\ln t + \begin{pmatrix}2\\5\end{pmatrix}t^{-1} - \begin{pmatrix}\frac{1}{2}\\0\end{pmatrix}t^{-2}$

6. $\mathbf{x} = c_1\begin{pmatrix}1\\2\end{pmatrix} + c_2\begin{pmatrix}-2\\1\end{pmatrix}e^{-5t} + \begin{pmatrix}1\\2\end{pmatrix}\ln t + \frac{8}{5}\begin{pmatrix}1\\2\end{pmatrix}t + \frac{4}{25}\begin{pmatrix}-2\\1\end{pmatrix}$

7. $\mathbf{x} = c_1\begin{pmatrix}1\\2\end{pmatrix}e^{3t} + c_2\begin{pmatrix}1\\-2\end{pmatrix}e^{-t} + \frac{1}{4}\begin{pmatrix}1\\-8\end{pmatrix}e^t$

8. $\mathbf{x} = c_1\begin{pmatrix}1\\1\end{pmatrix}e^t + c_2\begin{pmatrix}1\\3\end{pmatrix}e^{-t} + \begin{pmatrix}1\\0\end{pmatrix}e^t + 2\begin{pmatrix}1\\1\end{pmatrix}te^t$

9. $\mathbf{x} = c_1\begin{pmatrix}1\\1\end{pmatrix}e^{-t/2} + c_2\begin{pmatrix}1\\-1\end{pmatrix}e^{-2t} + \begin{pmatrix}\frac{5}{2}\\\frac{3}{2}\end{pmatrix}t - \begin{pmatrix}\frac{17}{4}\\\frac{15}{4}\end{pmatrix} + \begin{pmatrix}\frac{1}{6}\\\frac{1}{2}\end{pmatrix}e^t$

10. $\mathbf{x} = c_1\begin{pmatrix}1\\\sqrt{2}\end{pmatrix}e^{-t} + c_2\begin{pmatrix}-\sqrt{2}\\1\end{pmatrix}e^{-4t} - \frac{1}{3}\begin{pmatrix}\sqrt{2}-1\\2-\sqrt{2}\end{pmatrix}te^{-t} + \frac{1}{9}\begin{pmatrix}2+\sqrt{2}\\-1-\sqrt{2}\end{pmatrix}e^{-t}$

11. $\mathbf{x} = c_1\begin{pmatrix}5\cos t\\2\cos t + \operatorname{sen} t\end{pmatrix} + c_2\begin{pmatrix}5\operatorname{sen} t\\-\cos t + 2\operatorname{sen} t\end{pmatrix} + \begin{pmatrix}0\\1/2\end{pmatrix}t\cos t - \begin{pmatrix}5/2\\1\end{pmatrix}t\operatorname{sen} t - \begin{pmatrix}5/2\\1\end{pmatrix}\cos t$

12. $\mathbf{x} = [\,\frac{1}{5}\ln(\operatorname{sen} t) - \ln(-\cos t) - \frac{2}{5}t + c_1]\begin{pmatrix}5\cos t\\2\cos t + \operatorname{sen} t\end{pmatrix}$

13. **(a)** $\Psi(t) = \begin{pmatrix}e^{-t/2}\cos\frac{1}{2}t & e^{-t/2}\operatorname{sen}\frac{1}{2}t\\4e^{-t/2}\operatorname{sen}\frac{1}{2}t & -4e^{-t/2}\cos\frac{1}{2}t\end{pmatrix}$ **(b)** $\mathbf{x} = e^{-t/2}\begin{pmatrix}\operatorname{sen}\frac{1}{2}t\\4-4\cos\frac{1}{2}t\end{pmatrix}$

14. $\mathbf{x} = c_1\begin{pmatrix}1\\1\end{pmatrix}t + c_2\begin{pmatrix}1\\3\end{pmatrix}t^{-1} - \begin{pmatrix}2\\3\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\3\end{pmatrix}t - \begin{pmatrix}1\\1\end{pmatrix}t\ln t - \frac{1}{3}\begin{pmatrix}4\\3\end{pmatrix}t^2$

15. $\mathbf{x} = c_1\begin{pmatrix}2\\1\end{pmatrix}t^2 + c_2\begin{pmatrix}1\\2\end{pmatrix}t^{-1} + \begin{pmatrix}3\\2\end{pmatrix}t + \frac{1}{10}\begin{pmatrix}-2\\1\end{pmatrix}t^4 - \frac{1}{2}\begin{pmatrix}2\\3\end{pmatrix}$